Time Optimal Control of Dynamic Systems for Industry 4.0

Industry 4.0 School & Industry Night

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Agenda

- ❑ Process Industry Challenges and Solutions
- ❑ Industrial Robotics
- ❑ Introduction Optimal Control Problems
- ❑ Optimal Control Overview
- ❑ Time Optimal Control of Multi-DOF
- ❑ Industry Revolutionizing by Online Machine Learning
- ❑ Google Tools
- ❑ Conclusion and Biography

Process Industry Challenges

- Limitations in decision-making, production operation, efficiency and security, information integration, and so forth in process industry.

- To promote a vision of the process industry with efficient, green, and smart production, modern information technology should be utilized throughout the entire optimization process for production, management, and marketing.

Process Industry Solutions: Industry 4.0

- ✓ Intelligent sensing and integration of all process information, including production and management information.

- ✓ Collaborative decision-making in the supply chain, industry chain, and value chain, driven by knowledge;

- ✓ Using intelligent robotics and cooperative control and optimization of plant-wide production processes via human-cyber-physical interaction.

Industrial Robotics Challenges and Solutions

Introduction:

The main job of an industrial robot is to move an object on a pre-specified path, rest to rest, repeatedly.

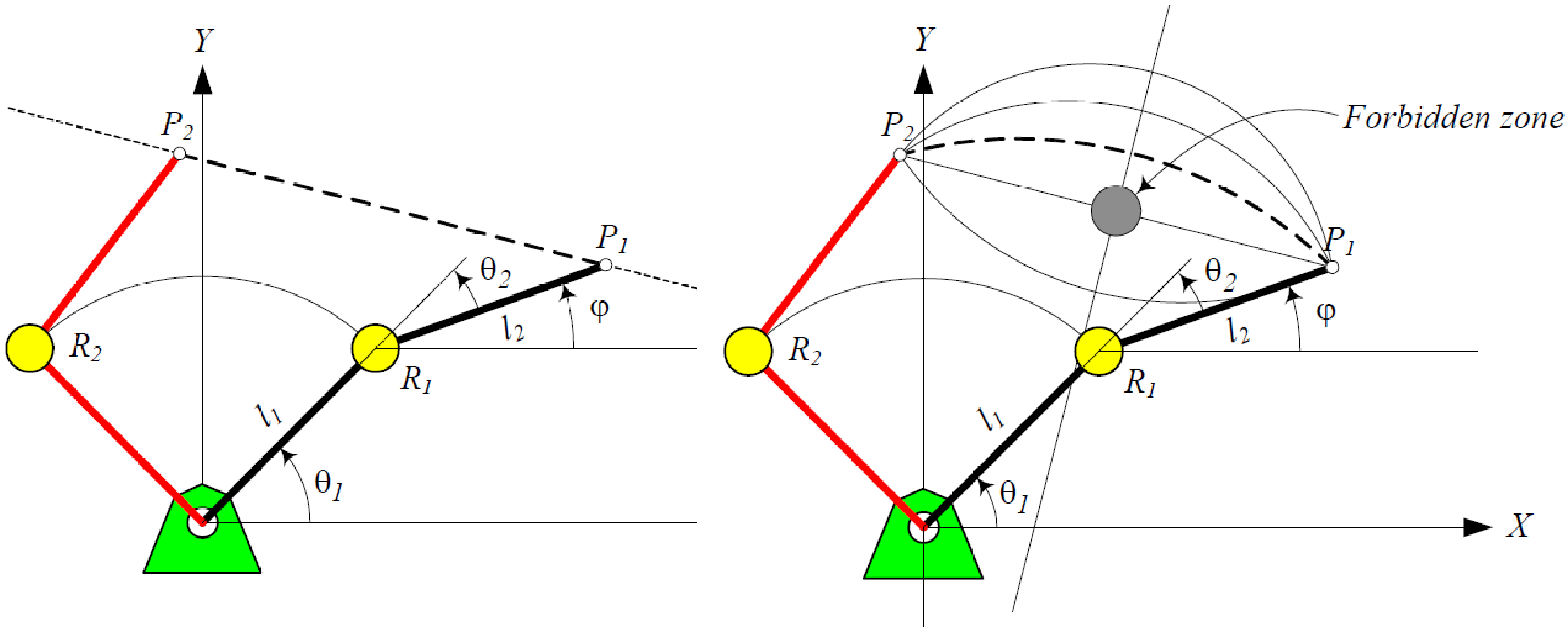
Challenges:

- ❑ Defined by a unilaterally constrained manipulator.
- ❑ To increase productivity, the robot should do its job in minimum time.

Solutions:

- ✓ Time optimal control of an n DOF dynamics system.
- ✓ A novel numerical method to solve the time optimal control problem of multi degree of freedom robots.
- ✓ Our approach explicitly takes into account inequality constraints and resulting contact forces as part of the system dynamics.
- ✓ To promote a vision of the process industry with Google tools

Introduction Optimal Control Problems



A 2R robot moveing along a given line

$$|\mathbf{Q}(t)| \leq \mathbf{Q}_{Max}$$

Optimal Control Overview

functional J $$J(x) = \int_{t_0}^{t_f} f(x, \dot{x}, t)dt$$

boundary conditions $$x(t_0) = x_0,\ x(t_f) = x_f$$

minimizing path $$x = x^{\star}(t)$$

for all continuous paths $x(t)$ $$J(x) \geq J(x^{\star})$$

$$x(t) = x^{\star} + \epsilon y(t) \qquad \epsilon \ll 1$$

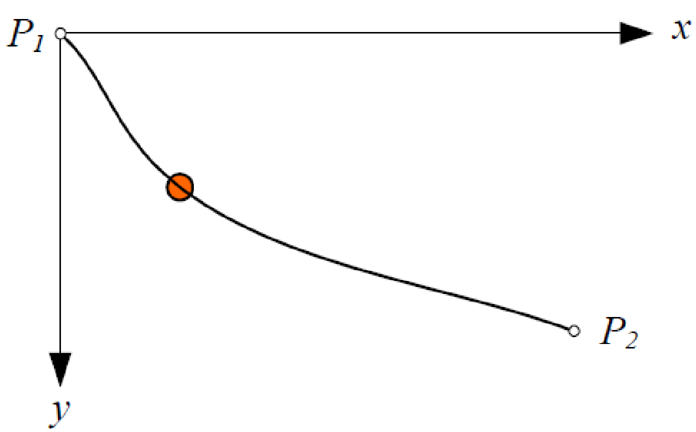
$$\Delta J = J\left(x^{\star} + \epsilon y(t)\right) - J\left(x^{\star}\right) = \epsilon V_1 + \epsilon^2 V_2 + O\left(\epsilon^3\right)$$

first variation of J $$V_1 = \int_{t_0}^{t_f} \left(y\frac{\partial f}{\partial x} + \dot{y}\frac{\partial f}{\partial \dot{x}} \right) dt$$

second variation of J $$V_2 = \int_{t_0}^{t_f} \left(y^2\frac{\partial^2 f}{\partial x^2} + 2y\dot{y}\frac{\partial^2 f}{\partial x\, \partial \dot{x}} + \dot{y}^2\frac{\partial^2 f}{\partial \dot{x}^2} \right)$$

Euler-Lagrange equation. $$\frac{\partial f}{\partial x} - \frac{d}{dt}\frac{\partial f}{\partial \dot{x}} = 0$$

Example: Brachistochron



A curve joining points P_1 and P_2, and a frictionless sliding point.

discussed by Galilei in 1630

solved by Bernoulli in 1696

$$J = \int_{t_0}^{t_f} dt \qquad J = \int_1^2 \frac{ds}{v}$$

$$v = \sqrt{2gy}. \qquad ds = \sqrt{1 + y'^2}dx$$

$$J = \int_1^2 \sqrt{\frac{1 + y'^2}{2gy}}dx$$

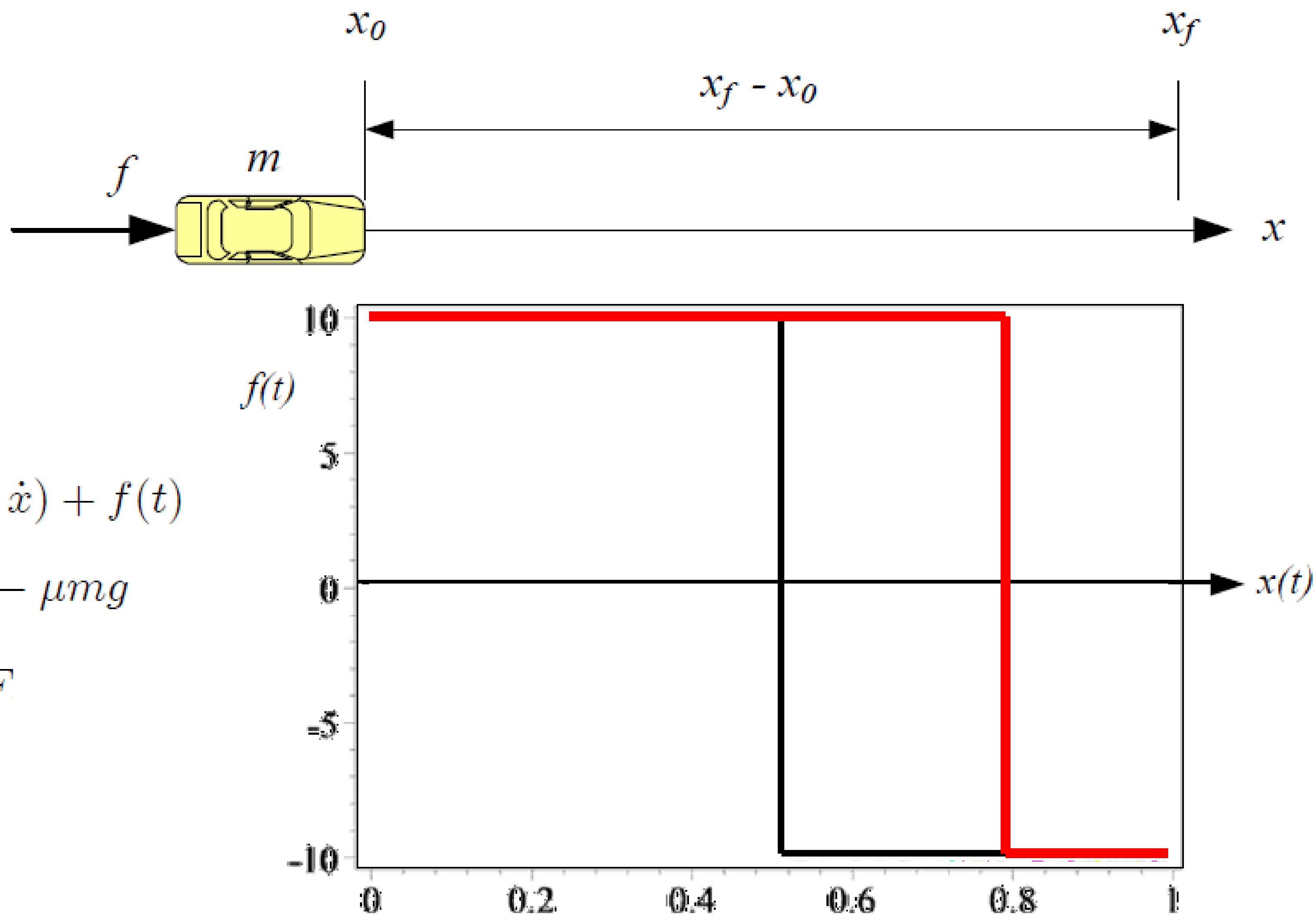
Applying the Lagrange equations

$$y\left(1 + y'^2\right) = 2r$$

The optimal curve is a ***cycloid***

$$\begin{aligned} x &= r\left(\beta - \sin\beta\right) \\ y &= r\left(1 - \cos\beta\right) \end{aligned}$$

Example: One DOF Dynamics System

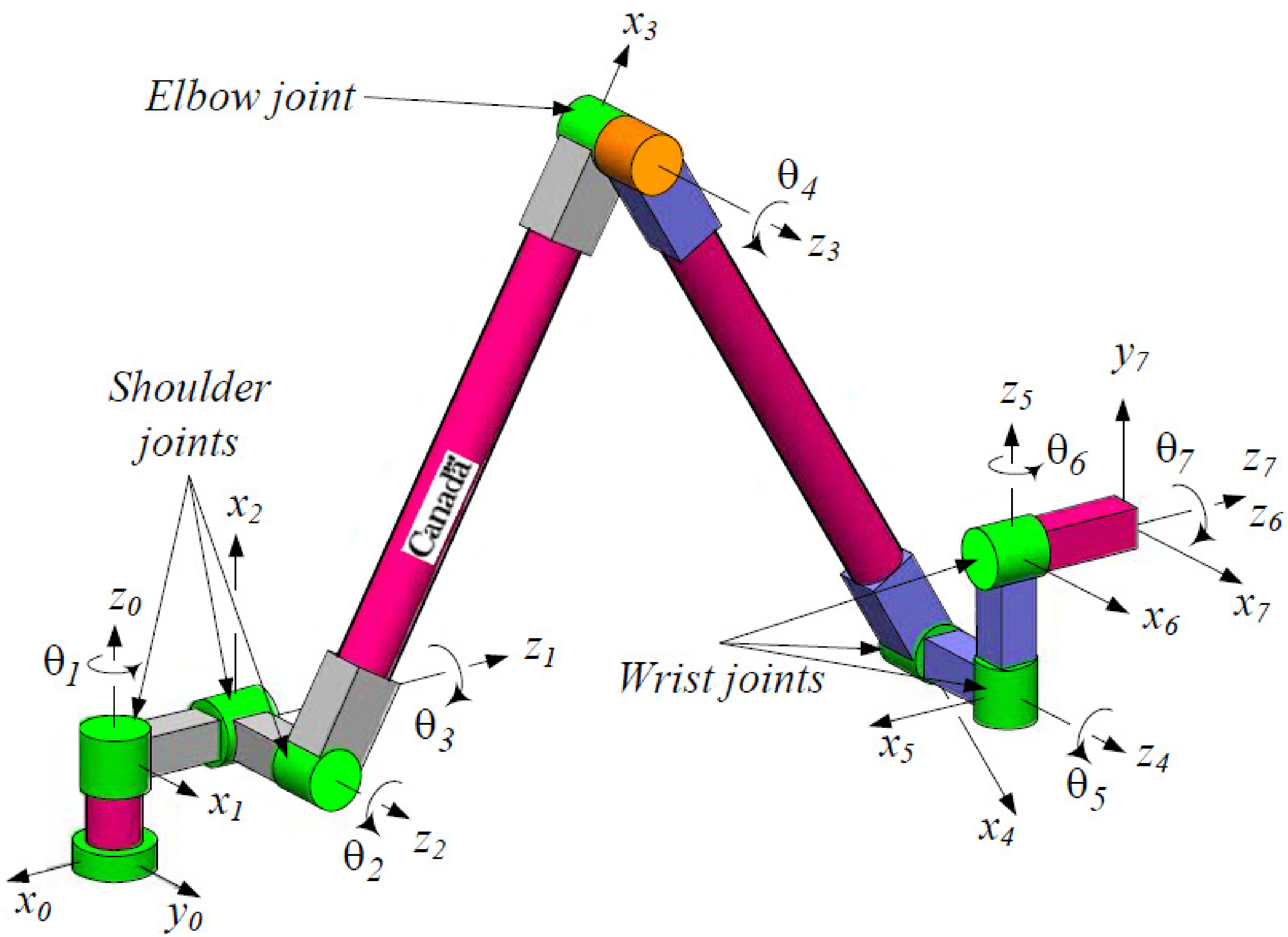


$$m\ddot{x} = g(x, \dot{x}) + f(t)$$

$$f = m\ddot{x} - \mu m g$$

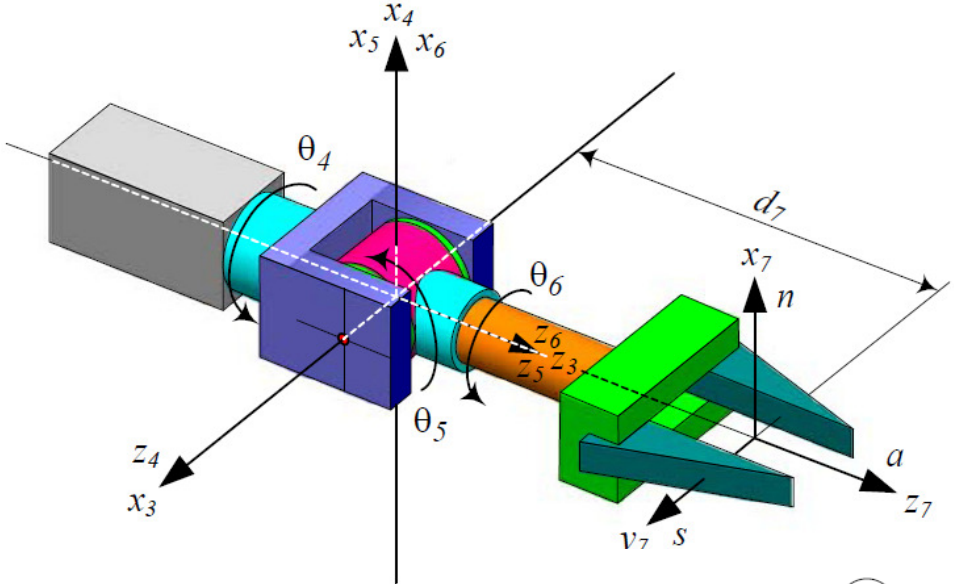
$$|f(t)| \leq F$$

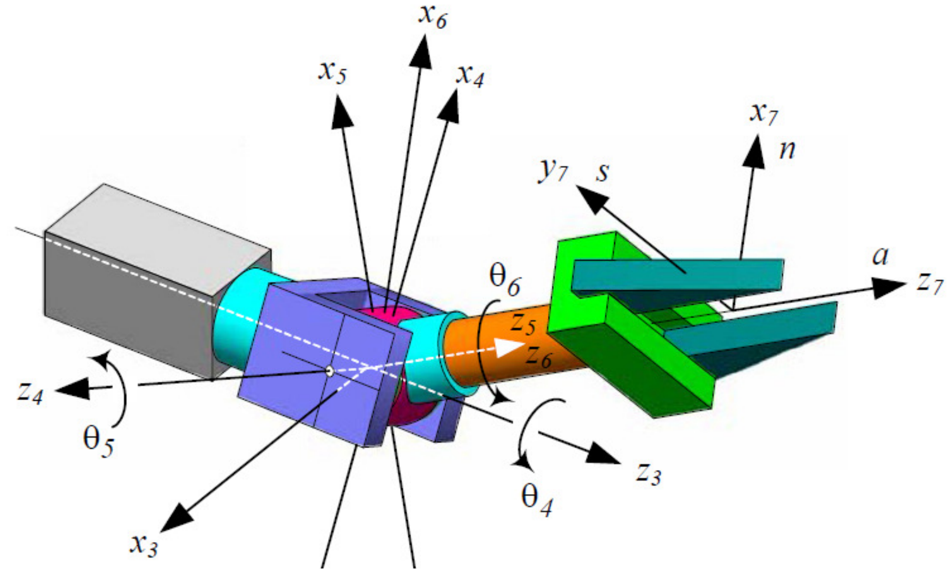
Example: Multi DOF Dynamics System

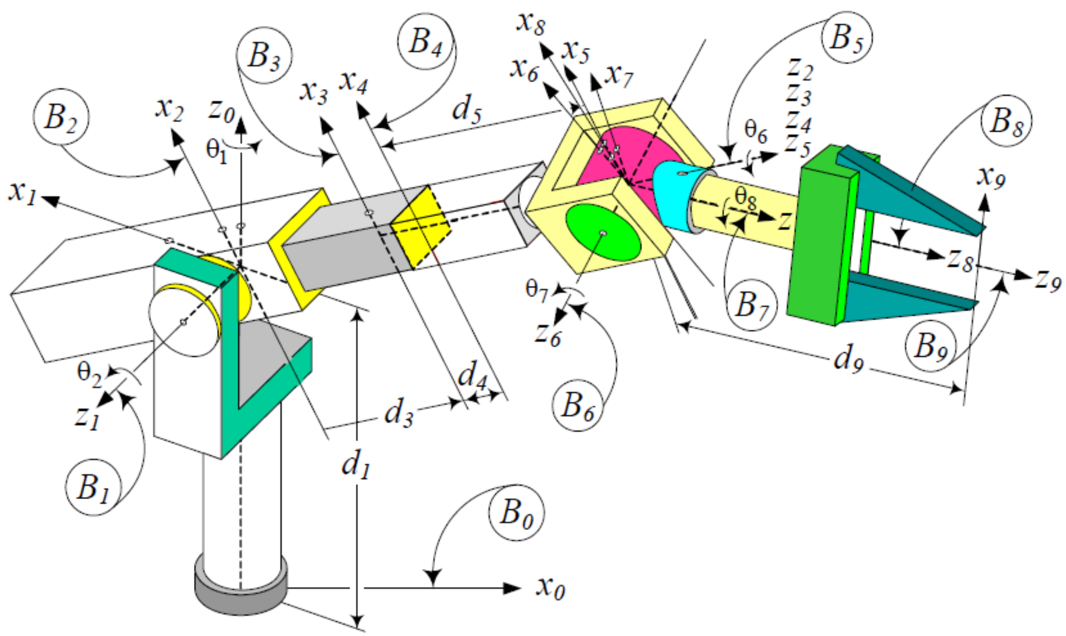


space station remote manipulator system

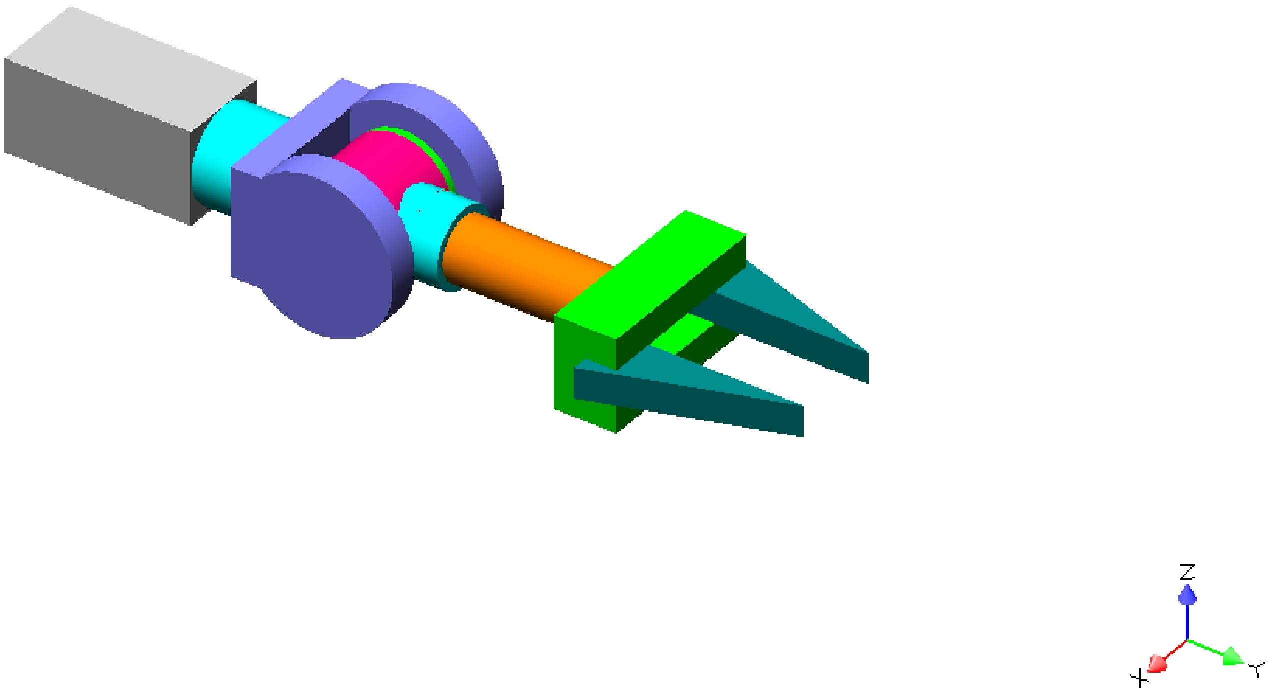
Example: Multi DOF Dynamics System



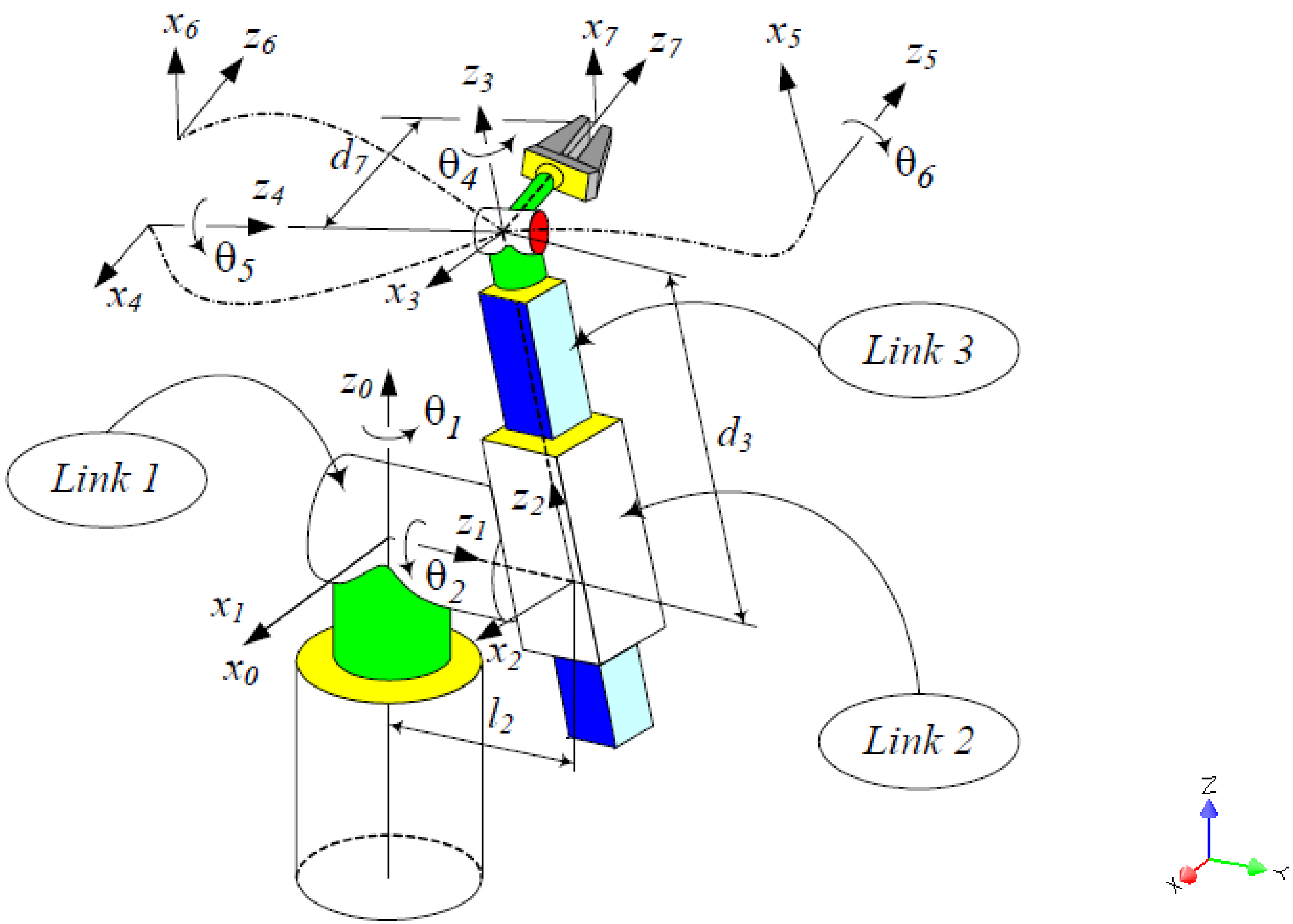




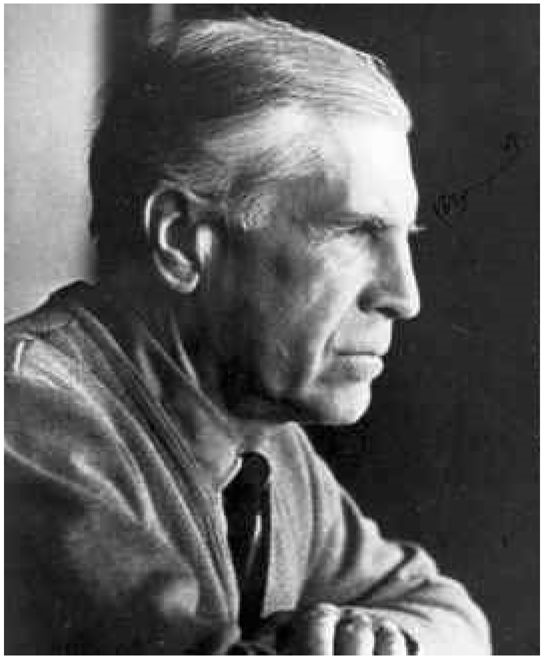
Example: Multi DOF Robotic Hand



Example: Multi DOF Stanford Arm



Time Optimal Control of Multi-DOF Dynamic Systems



Lev Semyonovich Pontryagin

(3 September 1908 – 3 May 1988)

Pontryagin principle:

the optimal control input vector ***Q***(t) to minimize the time of motion of a multi DOF dynamic system between two give states on a prescribed path with bounded input

$$|\mathbf{Q}(t)| \leq \mathbf{Q}_{Max}$$

is the one that at every instant has at least one component saturated. over the entire time interval.

$$\mathbf{Q}_{Max} \text{ or } -\mathbf{Q}_{Max}$$

Minimum Time and Bang-Bang Control

Consider a system with the following equation of motion:

$$\dot{\mathbf{x}} = \mathbf{f}\left(\mathbf{x}(t), \mathbf{Q}(t)\right) \tag{14.1}$$

where $\mathbf{Q}$ is the control input, and $\mathbf{x}$ is the state vector of the system.

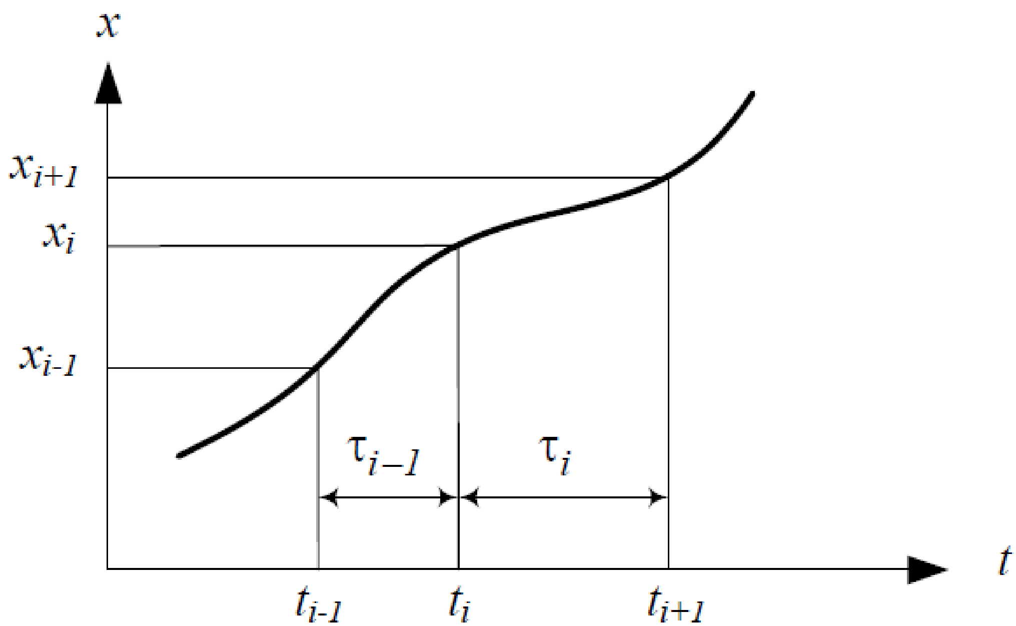
$$\mathbf{x} = \begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{bmatrix} \tag{14.2}$$

The minimum time problem is always subject to bounded input such as:

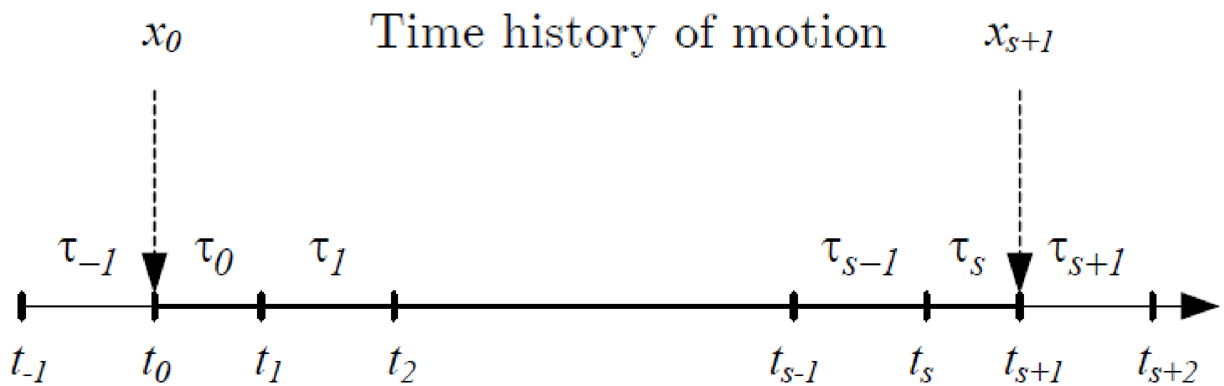
$$|\mathbf{Q}(t)| \leq \mathbf{Q}_{Max} \tag{14.3}$$

The solution of the time-optimal control problem subject to bounded input is *bang-bang control*. The control in which the input variable takes either the maximum or minimum values is called bang-bang control.

★ Floating Time Method

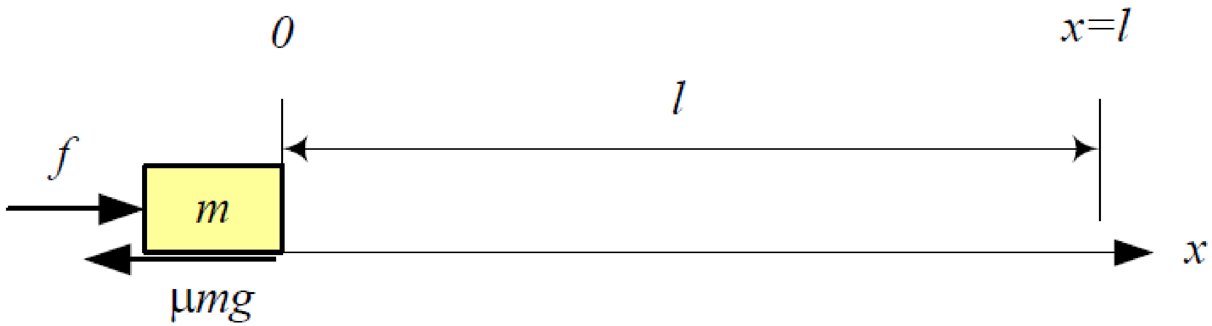


Time history of motion



Introducing two extra points, x_{-1} and x_{s+2}, before the initial and after the final points.

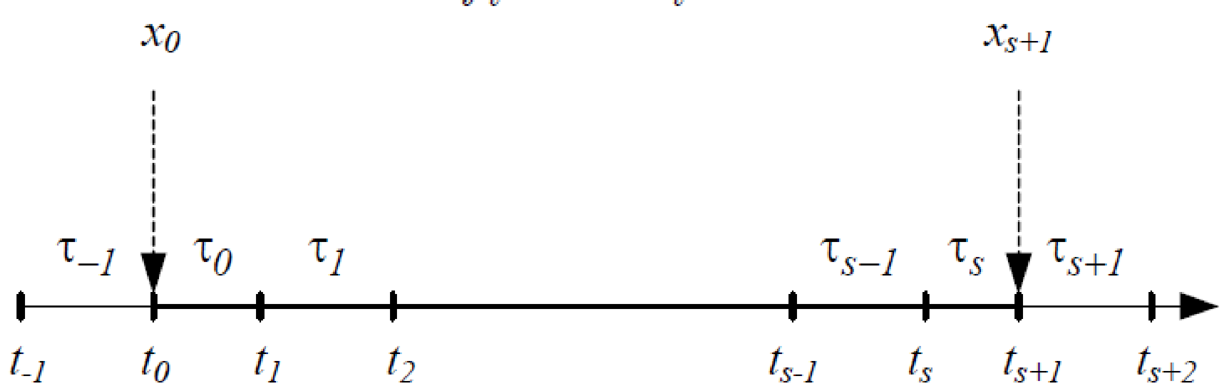
Example: One DOF Dynamics System



$$f = m\ddot{x} - \mu m g$$

$$|f(t)| \leq F.$$

$$f_i = m\ddot{x}_i$$



The rest conditions at the beginning

$$\dot{x}_i = \frac{x_{i+1} - x_{i-1}}{\tau_i + \tau_{i-1}}$$

$$x_{-1} = x_1 \qquad x_{s+2} = x_s$$

$$\tau_0 = \tau_{-1} \qquad \tau_{s+1} = \tau_s.$$

Example: One DOF Dynamics System

at the initial point

$$f_0 = m\ddot{x}_0 = \frac{4m}{2\tau_0^2}\left(x_1 - x_0\right)$$

The minimum value of the first floating time τ_0 is found by setting $f_0 = F$.

$$\tau_0 = \sqrt{\frac{2m\left(x_1 - x_0\right)}{F}}$$

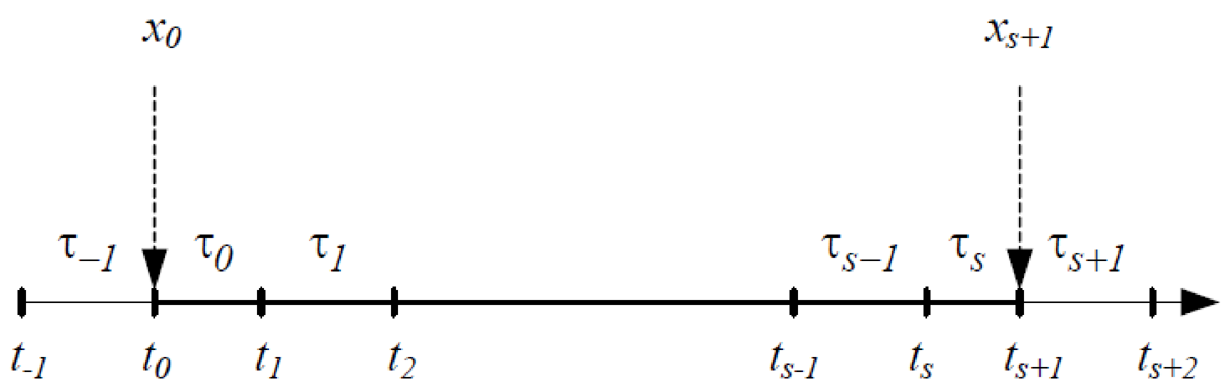
The minimum value of the final floating-time, τ_s, is achieved by $f_{s+1} = -F$.

$$\tau_s = \sqrt{\frac{2m\left(x_s - x_{s-1}\right)}{-F}}$$

To find the minimum value of τ_1

$$f_1 = \frac{4m}{\tau_1^2 + \tau_0^2}\left(\frac{\tau_0}{\tau_1 + \tau_0}x_2 + \frac{\tau_1}{\tau_1 + \tau_0}x_0 - x_1\right)$$

Example: One DOF Dynamics System



f_s can be found from the equation of motion at $i = s$ and substituting τ_s, τ_{s-1}, x_{s-1}, x_s, and x_{s+1}

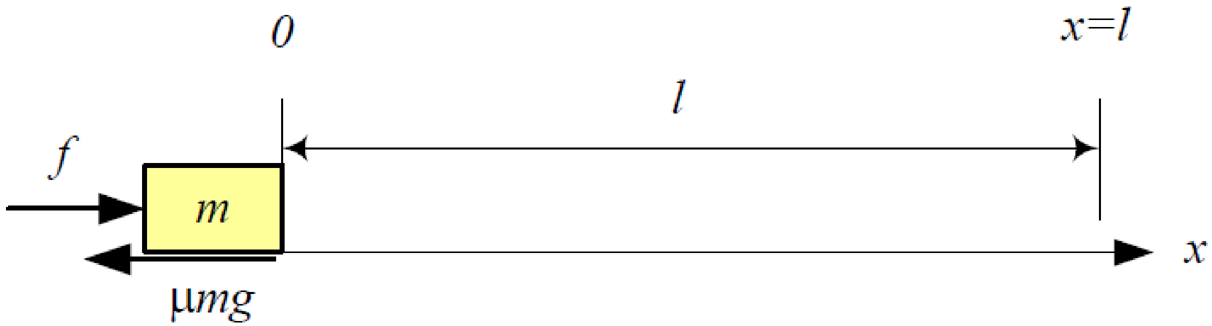
$$f_s = \frac{4m}{\tau_s^2 + \tau_{s-1}^2}\left(\frac{\tau_{s-1}}{\tau_s + \tau_{s-1}} x_{s+1} + \frac{\tau_s}{\tau_s + \tau_{s-1}} x_{s-1} - x_s\right)$$

if f_s does not break the constraint $|f(t)| \leq F$, the problem is solved

However, it is expected that f_s breaks the constraint $|f(t)| \leq F$

Now we reverse the procedure, and start a *backward path*.

Example: One DOF Dynamics System



$$f = m\ddot{x} - \mu m g$$

$$|f(t)| \leq F.$$

Rest-to-rest motion of a mass on a straight line time-optimally.

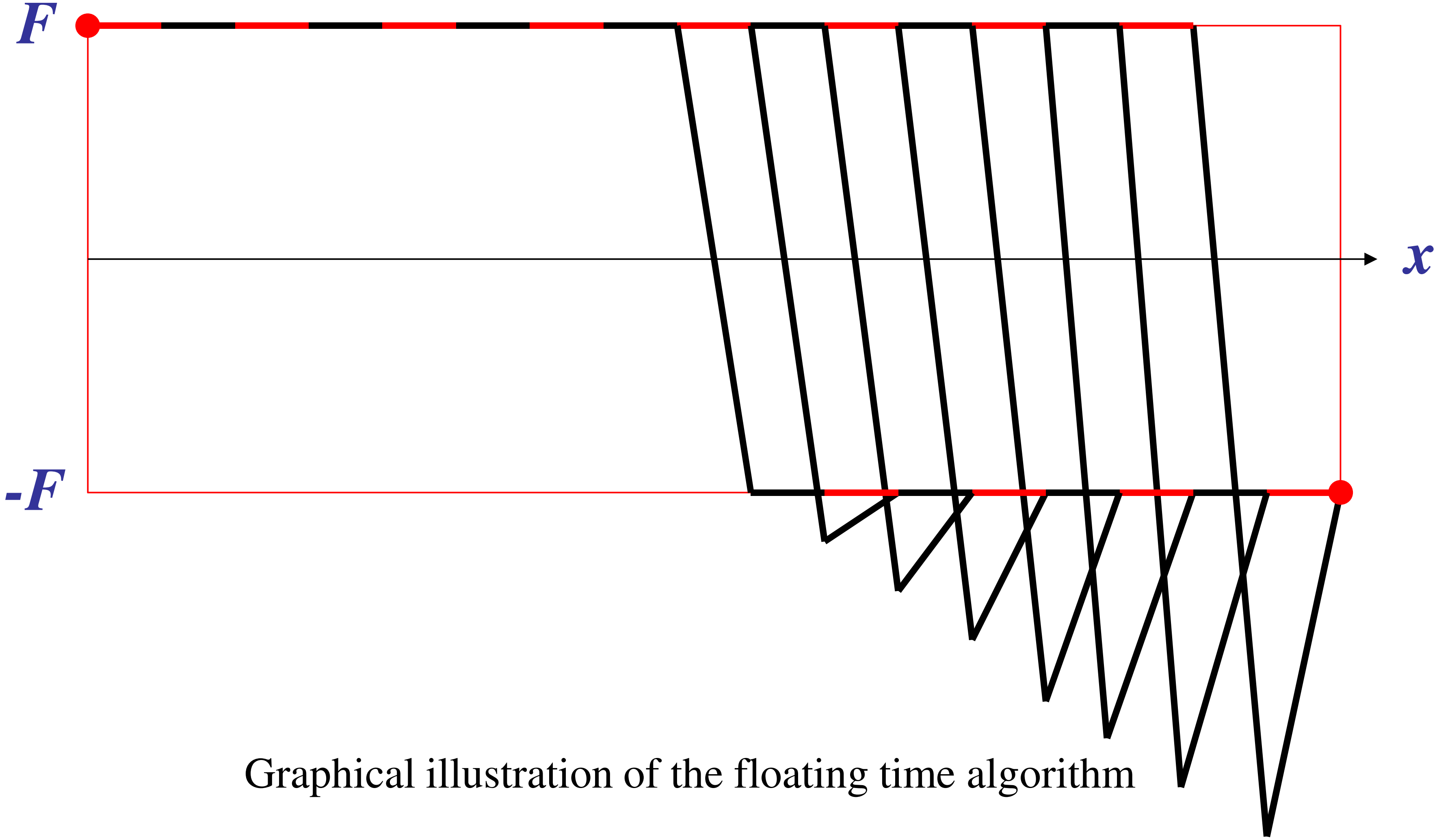
$$f_i = m\ddot{x}_i \qquad \begin{array}{ll} x(0) = 0 & v(0) = 0 \\ x(t_f) = l & v(t_f) = 0. \end{array}$$

$$F = 10\,\mathrm{N} \qquad l = 1 \quad \mathrm{m} \qquad s + 1 = 200$$

for $\mu = 0$ *the switching point* $t = \tau = t_f/2$ *and*

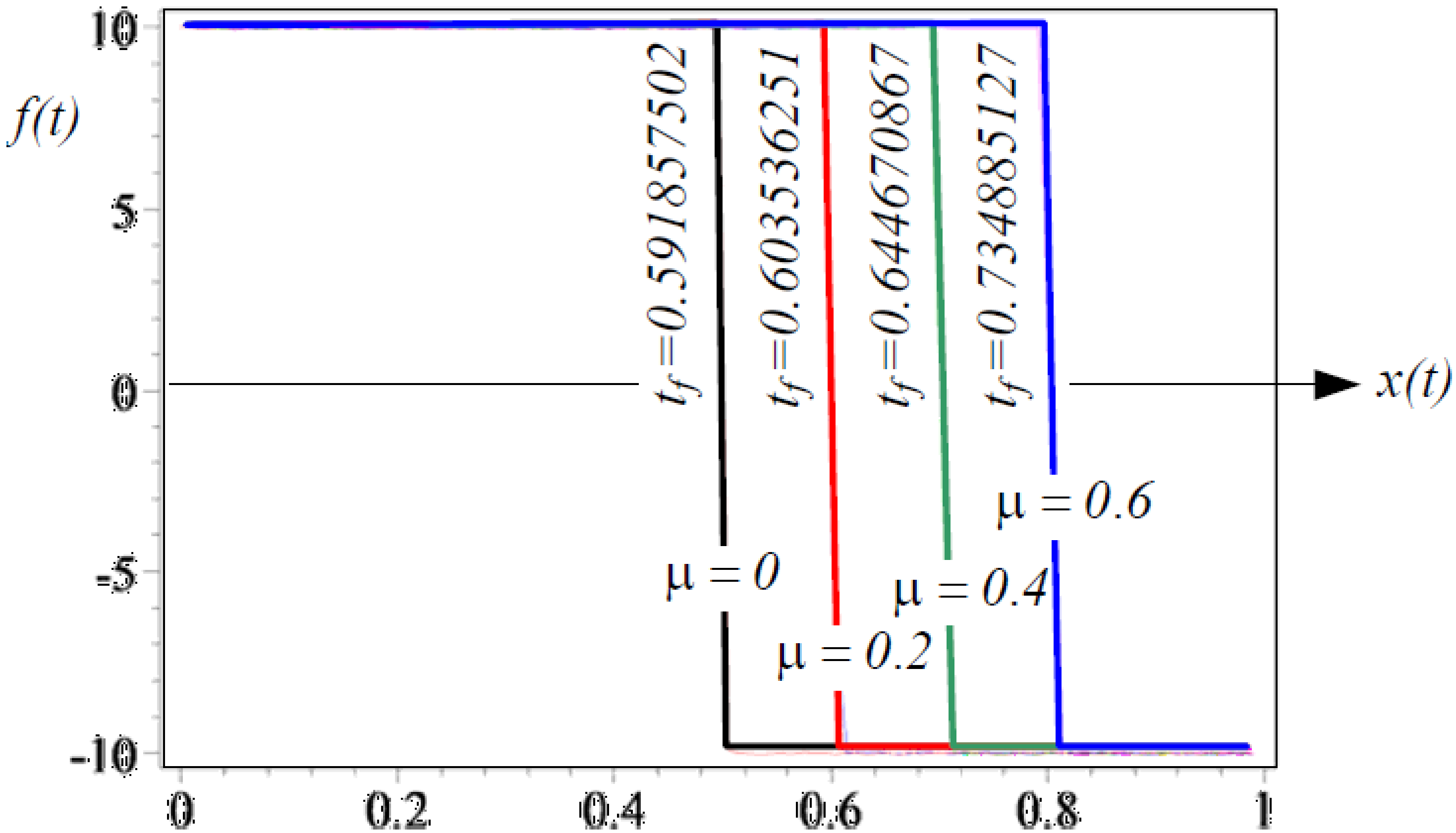
$$f(t) = \begin{cases} F & if \quad t < \tau \\ -F & if \quad t > \tau. \end{cases}$$

Example: One DOF Dynamics System



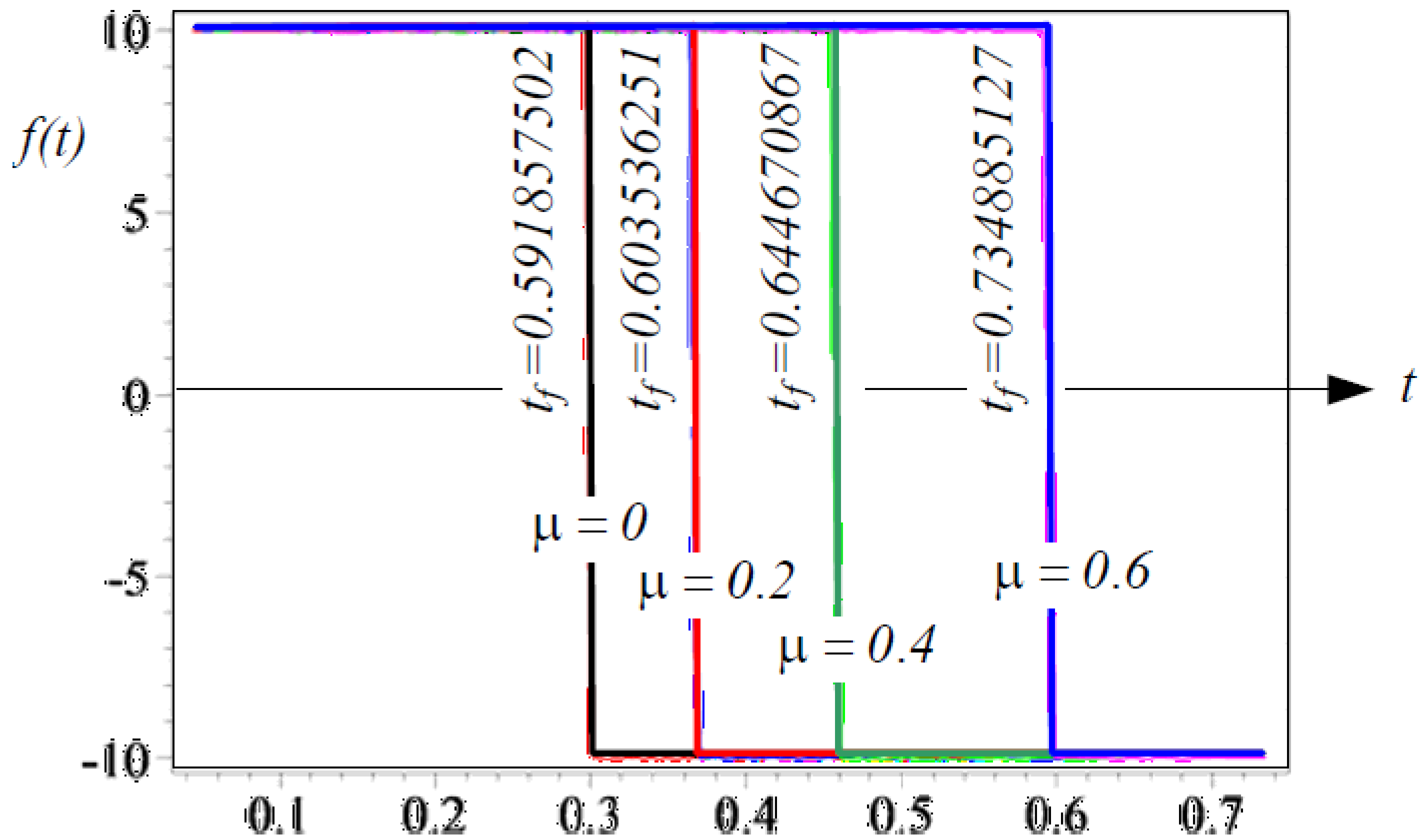
Graphical illustration of the floating time algorithm

Example: One DOF Dynamics System



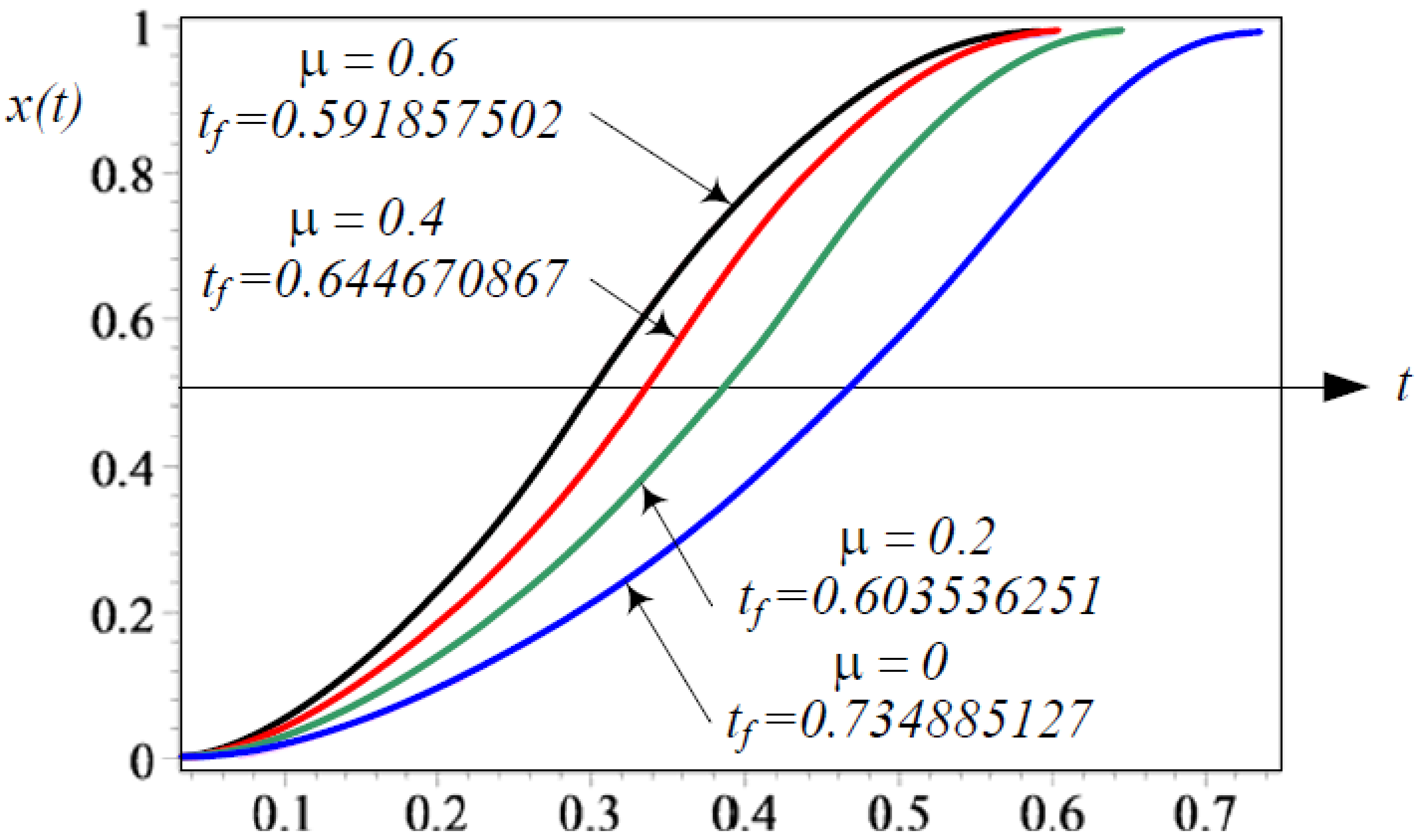
Position history of the optimal force $f(t)$ for different friction coefficients μ.

Example: One DOF Dynamics System



Time history of the optimal input $f(t)$ for different friction coefficients μ

Example: One DOF Dynamics System



Time history of the optimal motion $x(t)$ for different friction coefficients μ.

★ Floating Time Method

Algorithm

1. *Divide the preplanned path of motion* $x(t)$ *into* $s+1$ *intervals and specify all coordinate values* x_i, $(i = 0, 1, 2, 3, ..., s+1)$

2. *Set* $f_0 = +F$ *and calculate* $$\tau_0 = \sqrt{\frac{2m(x_1 - x_0)}{F}}$$

3. *Set* $f_{s+1} = -F$ *and calculate* $$\tau_s = \sqrt{\frac{2m(x_s - x_{s-1})}{-F}}$$

4. *For* i *from* 1 *to* $s-1$, *calculate* τ_i *such that* $f_i = +F$ and

$$\begin{aligned} f_i &= m\ddot{x}_i - g(x_i, \dot{x}_i) \\ &= \frac{4m}{\tau_i^2 + \tau_{i-1}^2}\left(\frac{\tau_{i-1}}{\tau_i + \tau_{i-1}}x_{i+1} + \frac{\tau_i}{\tau_i + \tau_{i-1}}x_{i-1} + x_i\right) - g(x_i, \dot{x}_i) \end{aligned}$$

5. *If* $|f_i| \leq F$, *then stop, otherwise set* $j = s$,

6. *Calculate* τ_{j-1} *such that* $f_j = -F$

7. *If* $|f_{j-1}| \leq F$, *then stop, otherwise set* $j = j-1$ *and return to step 6*

★ Convergence.

τ_i *converges to the minimum possible value, as long as*

$$\partial \ddot{x}_i / \partial \tau_i < 0 \quad \text{and} \quad \partial \ddot{x}_i / \partial \tau_{i-1} > 0$$

$$\begin{aligned} Z_1 x_{s+1} + Z_2 x_s + Z_3 x_{s-1} &< 0 \\ Z_4 x_{s+1} + Z_5 x_s + Z_6 x_{s-1} &> 0 \end{aligned}$$

for backward path

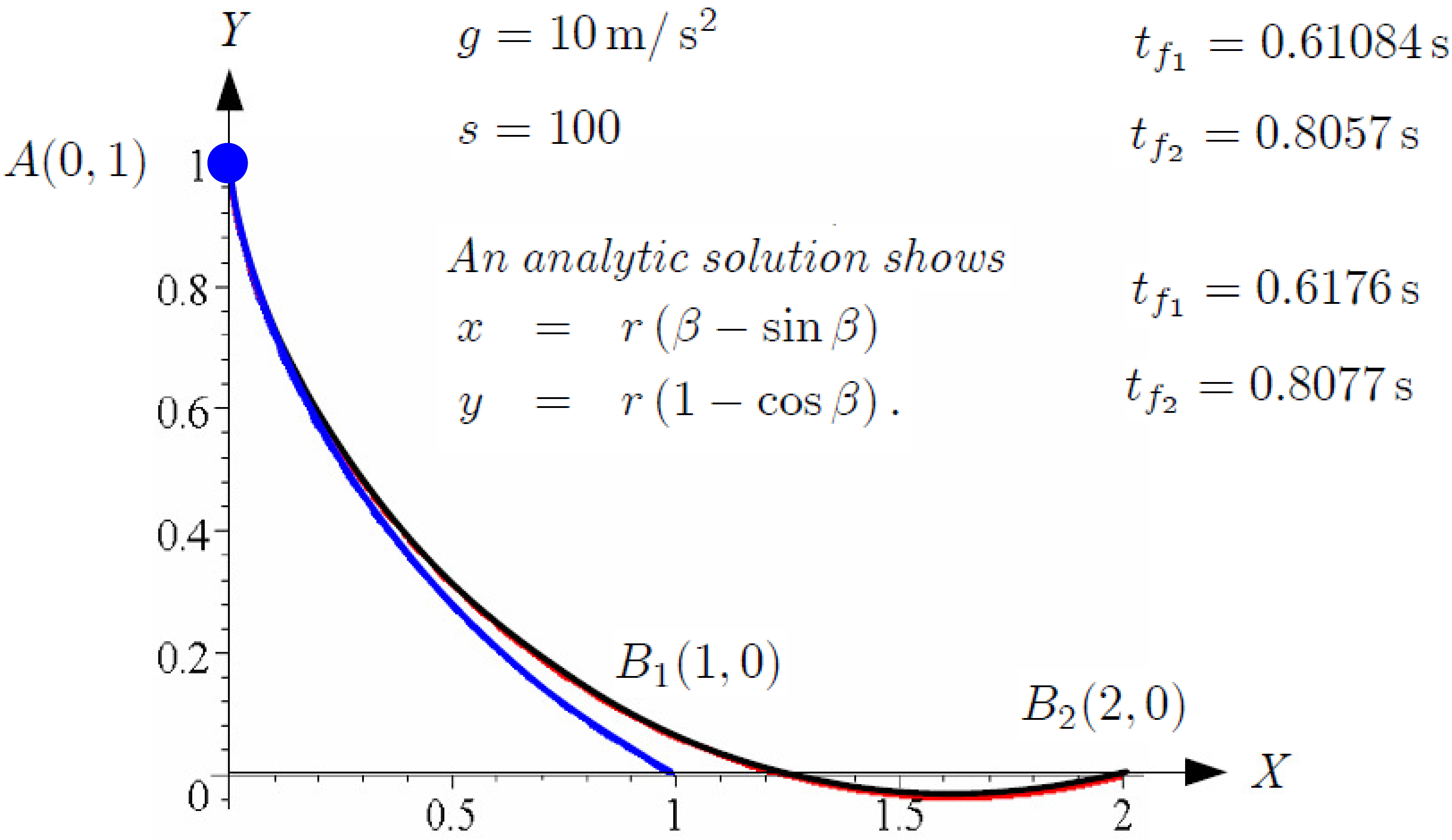
$$\begin{aligned} Z_1 x_{s+1} + Z_2 x_s + Z_3 x_{s-1} &> 0 \\ Z_4 x_{s+1} + Z_5 x_s + Z_6 x_{s-1} &< 0 \end{aligned}$$

termination criterion

$$||f_i| - F| \leq \epsilon.$$

$$\begin{aligned}
Z_1 &= \frac{8\left(6\tau_i^4\tau_{i-1} + 8\tau_i^3\tau_{i-1}^2 + 6\tau_i^2\tau_{i-1}^3\right)}{\left(\tau_i^2 + \tau_{i-1}^2\right)^3\left(\tau_i + \tau_{i-1}\right)^3} \\
Z_2 &= \frac{8\left(\tau_{i-1}^5 - 3\tau_i^5 - 8\tau_i^3\tau_{i-1}^2 - 9\tau_i^4\tau_{i-1} + 3\tau_i\tau_{i-1}^4\right)}{\left(\tau_i^2 + \tau_{i-1}^2\right)^3\left(\tau_i + \tau_{i-1}\right)^3} \\
Z_3 &= \frac{8\left(-\tau_{i-1}^5 + 3\tau_i^5 + 3\tau_i^4\tau_{i-1} - 3\tau_i\tau_{i-1}^4 - 6\tau_i^2\tau_{i-1}^3\right)}{\left(\tau_i^2 + \tau_{i-1}^2\right)^3\left(\tau_i + \tau_{i-1}\right)^3} \\
Z_4 &= \frac{8\left(3\tau_{i-1}^5 - \tau_i^5 - 6\tau_i^3\tau_{i-1}^2 - 3\tau_i^4\tau_{i-1} + 3\tau_i\tau_{i-1}^4\right)}{\left(\tau_i^2 + \tau_{i-1}^2\right)^3\left(\tau_i + \tau_{i-1}\right)^3} \\
Z_5 &= \frac{8\left(-3\tau_{i-1}^5 + \tau_i^5 + 3\tau_i^4\tau_{i-1} - 9\tau_i\tau_{i-1}^4 - 8\tau_i^2\tau_{i-1}^3\right)}{\left(\tau_i^2 + \tau_{i-1}^2\right)^3\left(\tau_i + \tau_{i-1}\right)^3} \\
Z_6 &= \frac{8\left(6\tau_i^3\tau_{i-1}^2 + 8\tau_i^2\tau_{i-1}^3 + 6\tau_i\tau_{i-1}^4\right)}{\left(\tau_i^2 + \tau_{i-1}^2\right)^3\left(\tau_i + \tau_{i-1}\right)^3}.
\end{aligned}$$

Example: Beachistochrone



Time optimal path for a falling unit mass from $A(0,1)$ to two different destinations.

Floating time technique for the n *DOF* systems.

1. *Divide the preplanned path of motion* $\mathbf{x}(t)$ *into* $s+1$ *intervals and specify all coordinate vectors* $\mathbf{x}_i$, $(i = 0, 1, 2, 3, ..., s+1)$.

2. *Develop the equations of motion at* $\mathbf{x}_0$ *and calculate* τ_0 *for which only one component of the force vector* $\mathbf{f}_0$ *is saturated on its higher limit, while all the other components are within their limits.*

$$\begin{array}{llll} f_{0_k} = F_k & , & k \in \{0, 1, 2, \cdots, n\} & \\ f_{0_r} \leq F_r & , & r = 0, 1, 2, \cdots, n & , \quad r \neq k \end{array}$$

3. *Develop the equations of motion at* $\mathbf{x}_{s+1}$ *and calculate* τ_s for which only one component of the force vector $\mathbf{f}_{s+1}$ *is saturated on its higher limit*, while all the other components are within their limits.

$$\begin{array}{llll} f_{s+1_k} = -F_k & , & k \in \{0, 1, 2, \cdots, n\} & \\ f_{s+1_r} \leq F_r & , & r = 0, 1, 2, \cdots, n & , \quad r \neq k \end{array}$$

Floating time technique for the n DOF systems.

4. *For i from 1 to $s-1$, calculate τ_i such that only one component of the force vector* $\mathbf{f}_i$ is saturated on its higher limit, *while all the other components are within their limits.*

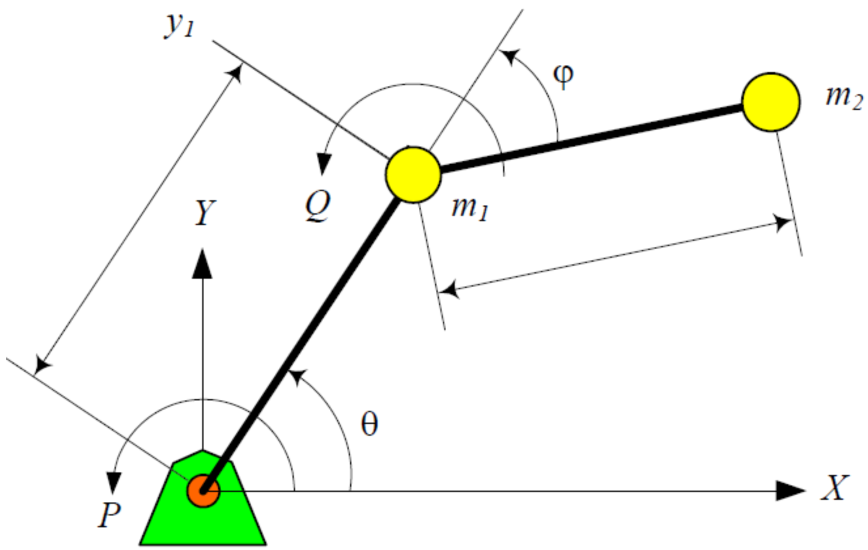
$$
\begin{aligned}
&f_{i_k} = -F_k \quad , \quad k \in \{0, 1, 2, \cdots, n\} \\
&f_{i_r} \leq F_r \quad , \quad r = 0, 1, 2, \cdots, n \quad , \quad r \neq k
\end{aligned}
$$

5. *If $|\mathbf{f}_s| \leq \mathbf{F}$, then stop, otherwise set $j = s$.*

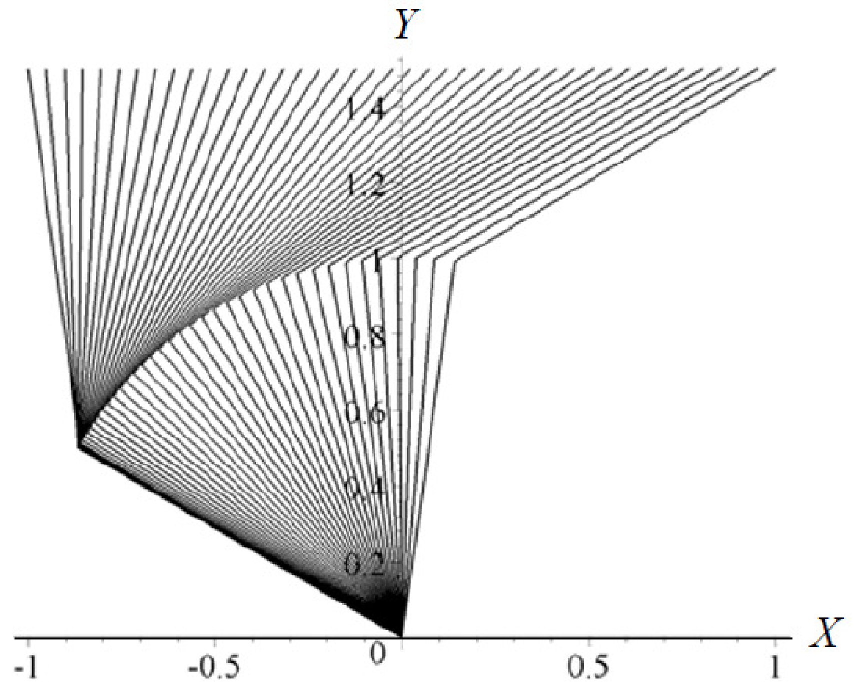
6. *Calculate τ_{j-1} such that only one component of the force vector $\mathbf{f}_j$ is saturated on its lower limit, while all the other components are within their limits.*

7. *If $|\mathbf{f}_{j-1}| \leq F$, then stop, otherwise set $j = j-1$ and return to step 6.*

★ $2R$ *manipulator on a straight line.*



A 2R planar manipulator with rigid arms



A 2R planar manipulator, moving from point $(1, 1.5)$ to point $(-1, 1.5)$ on a straight line $Y = 1.5$.

equations of motion for 2R robotic manipulators

$$\begin{aligned} P &= A\ddot{\theta} + B\ddot{\varphi} + C\dot{\theta}\dot{\varphi} + D\dot{\varphi}^2 + M \\ Q &= E\ddot{\theta} + F\ddot{\varphi} + G\dot{\theta}^2 + N \end{aligned}$$

where P and Q are the actuator torques and

$$\begin{aligned} A &= A(\varphi) = m_1 l_1^2 + m_2 \left(l_1^2 + l_2^2 + 2 l_1 l_2 \cos\varphi\right) \\ B &= B(\varphi) = m_2 \left(l_2^2 + l_1 l_2 \cos\varphi\right) \\ C &= C(\varphi) = -2 m_2 l_1 l_2 \sin\varphi \\ D &= D(\varphi) = m_2 l_1 l_2 \sin\varphi \\ E &= E(\varphi) = B \\ F &= m_2 l_2^2 \\ G &= G(\varphi) = -D \\ M &= M(\theta,\varphi) = (m_1 + m_2) g l_1 \cos\theta + m_2 g l_2 \cos(\theta + \varphi) \\ N &= N(\theta,\varphi) = m_2 g l_2 \cos(\theta + \varphi)\,. \end{aligned}$$

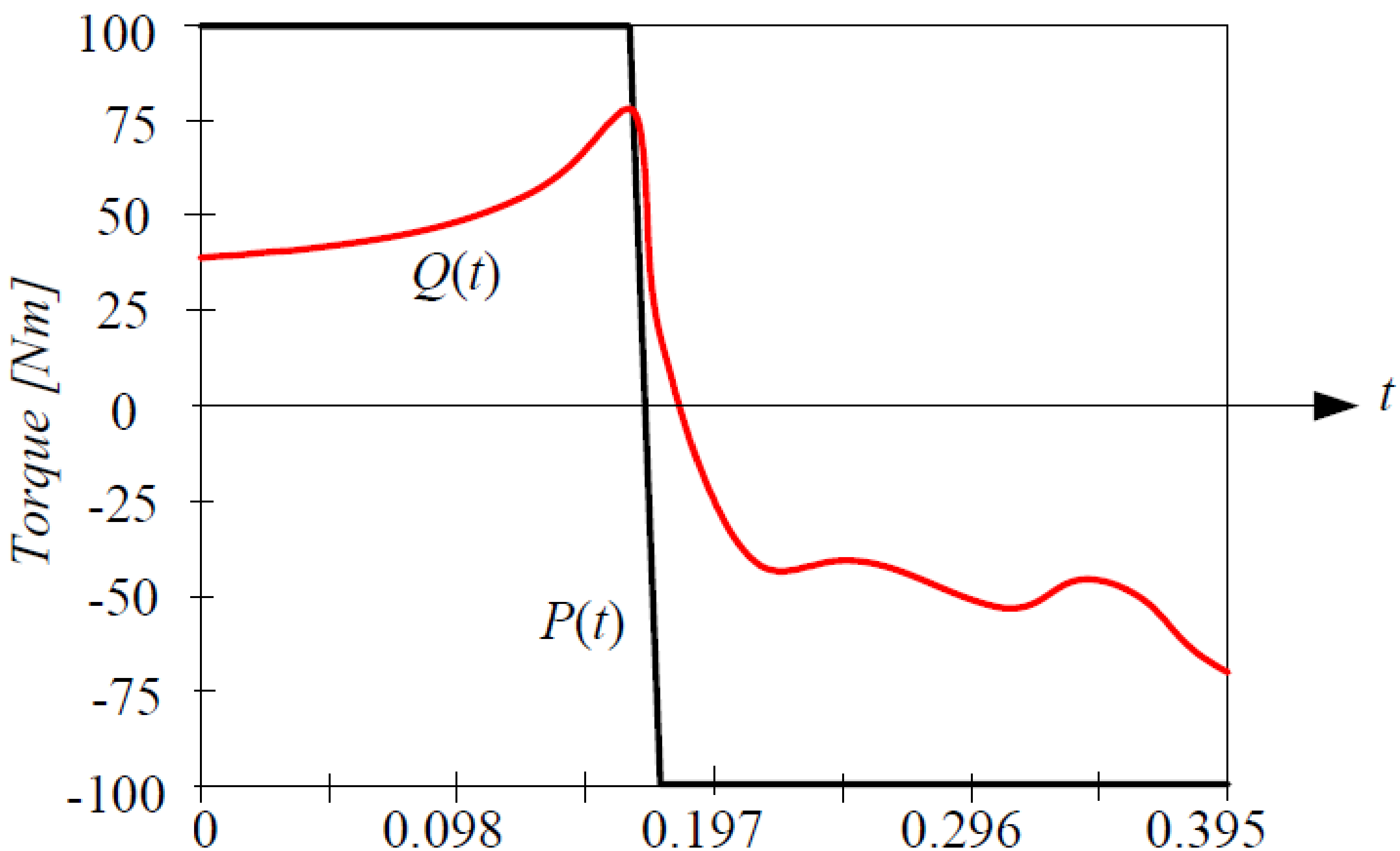
$$
\begin{aligned}
v(\dot{x}_i) &= \frac{x_{i+1} - x_{i-1}}{\tau_i + \tau_{i-1}} \\
a(\ddot{x}_i) &= \frac{4}{\tau_i^2 + \tau_{i-1}^2} \left(\frac{\tau_{i-1}}{\tau_i + \tau_{i-1}} x_{i+1} + \frac{\tau_i}{\tau_i + \tau_{i-1}} x_{i-1} - x_i \right)
\end{aligned}
$$

$$
\begin{aligned}
P_i(t) &= A_i a(\ddot{\theta}) + B_i a\left(\ddot{\varphi}\right) + C_i v(\dot{\theta}) v\left(\dot{\varphi}\right) + D_i v^2\left(\dot{\varphi}\right) + M_i \\
Q_i(t) &= E_i a(\ddot{\theta}) + F_i a\left(\ddot{\varphi}\right) + G_i v^2(\dot{\theta}) + N_i
\end{aligned}
$$

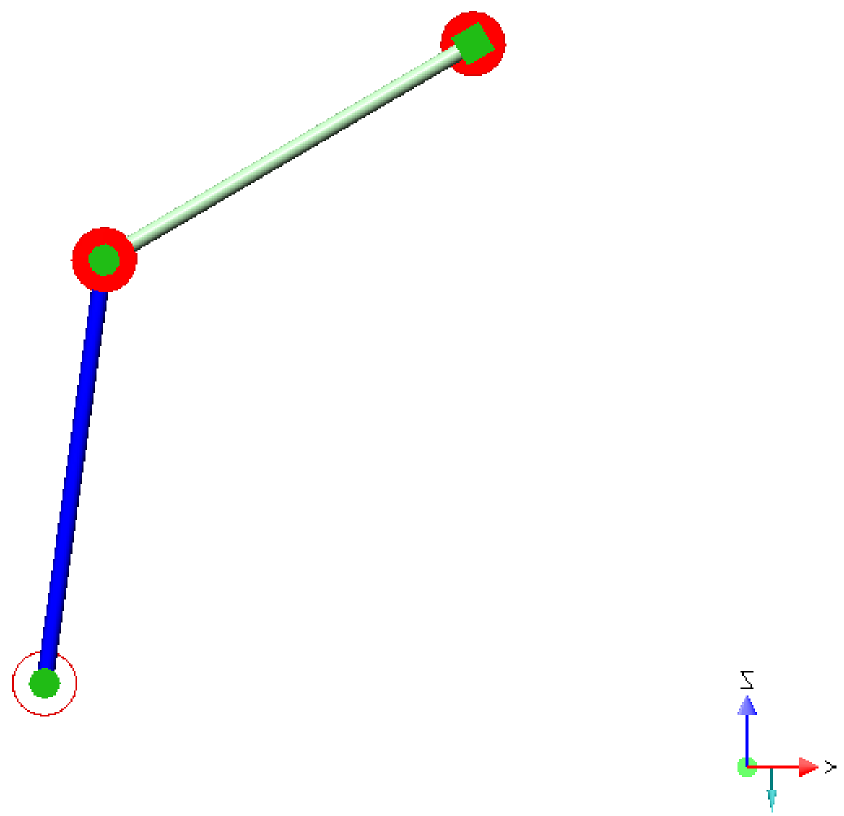
$$
|P_i(t)| \leq P_M \qquad |Q_i(t)| \leq Q_M.
$$

$$
\begin{aligned}
&\tau_0 = \tau_1 && \theta_0 = \theta_2 && \varphi_0 = \varphi_2 \\
&\tau_s = \tau_{s+1} && \theta_s = \theta_{s+2} && \varphi_s = \varphi_{s+2}
\end{aligned}
$$

$$
m_1 = m_2 = 1\,\mathrm{kg} \quad l_1 = l_2 = 1\,\mathrm{m} \quad P_M = Q_M = 100\,\mathrm{N\,m}
$$



Time optimal control inputs for a 2R manipulator moving on line $Y = 1.5$, $-1 < X < 1$



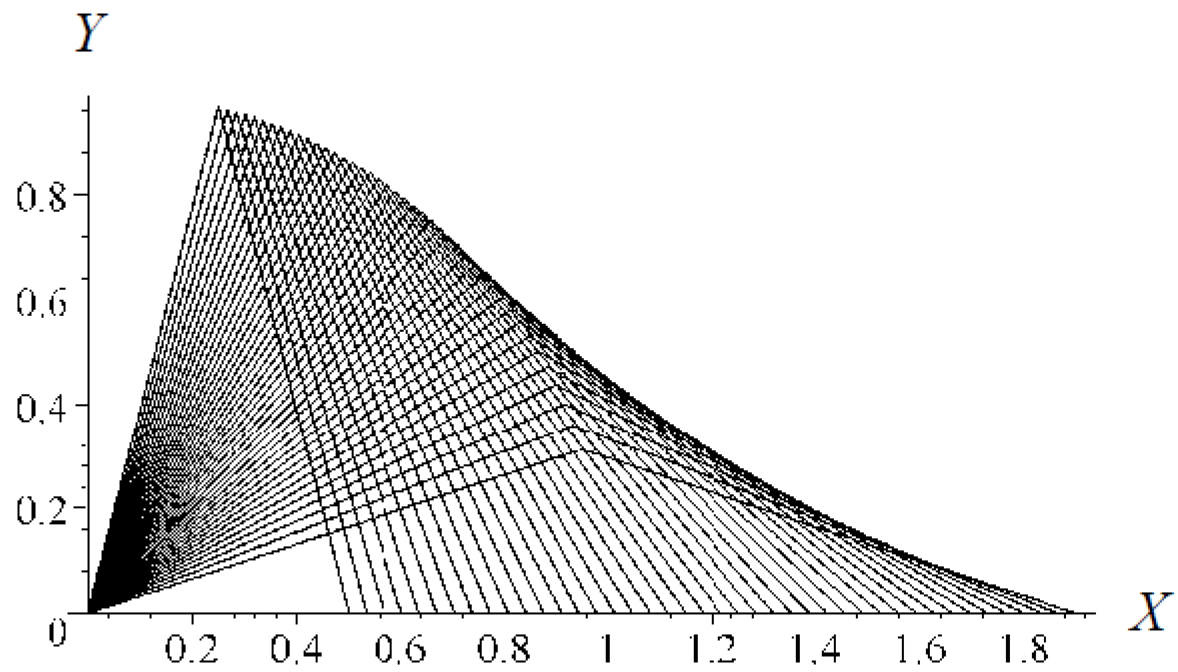
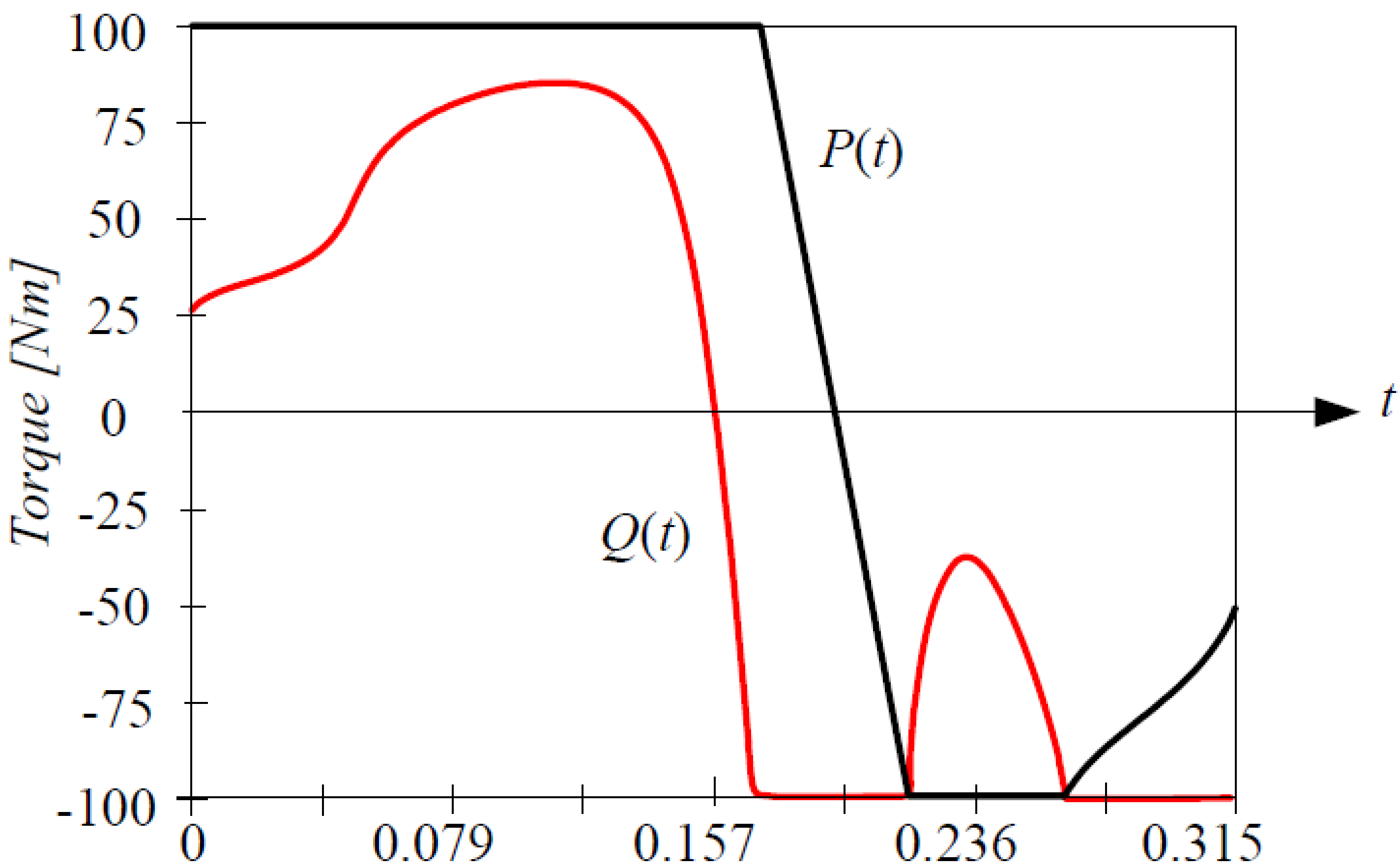


Illustration of motion of a 2R planar manipulator on line $y = 0$.

$$m_1 = m_2 = 1\,\mathrm{kg} \quad l_1 = l_2 = 1\,\mathrm{m} \quad P_M = Q_M = 100\,\mathrm{N\,m}$$
$$X(0) = 1.9\,\mathrm{m} \quad X(t_f) = 0.5\,\mathrm{m} \quad Y = 0$$



Time optimal control inputs for a 2R manipulator moving on line $Y = 0$, $0.5 < X < 1.9$

Conclusion

Time optimal control of an n DOF dynamics system solution:

- ✓ At every instant of time, at least one actuator must be saturated while the others are within their limits.
- ✓ Floating-time method is to find the saturated actuator, the switching points, and the output of the non-saturated actuators.
- ✓ The floating-time method is based on discrete equations of motion, utilizing variable time increments.
- ✓ Smart equipment such as intelligent robotics in manufacturing processes,   increase productivity, the robot should do its job in minimum time.

Biography of Presenter

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Thank You