Time Optimal Control of Dynamic Systems for Industry 4.0

Industry 4.0 School & Industry Night
UBC Okanagan School of Engineering

Prof Reza N. Jazar
Email: reza.Jazar@rmit.edu.au
Agenda

- Process Industry Challenges and Solutions
- Industrial Robotics
- Introduction Optimal Control Problems
- Optimal Control Overview
- Time Optimal Control of Multi-DOF
- Industry Revolutionizing by Online Machine Learning
- Google Tools
- Conclusion and Biography
Limitations in decision-making, production operation, efficiency and security, information integration, and so forth in process industry.

To promote a vision of the process industry with efficient, green, and smart production, modern information technology should be utilized throughout the entire optimization process for production, management, and marketing.
Intelligent sensing and integration of all process information, including production and management information.

Collaborative decision-making in the supply chain, industry chain, and value chain, driven by knowledge;

Using intelligent robotics and cooperative control and optimization of plant-wide production processes via human-cyber-physical interaction.
Industrial Robotics Challenges and Solutions

Introduction:
The main job of an industrial robot is to move an object on a pre-specified path, rest to rest, repeatedly.

Challenges:
- Defined by a unilaterally constrained manipulator.
- To increase productivity, the robot should do its job in minimum time.

Solutions:
- Time optimal control of an n DOF dynamics system.
- A novel numerical method to solve the time optimal control problem of multi degree of freedom robots.
- Our approach explicitly takes into account inequality constraints and resulting contact forces as part of the system dynamics.
- To promote a vision of the process industry with Google tools.
Introduction Optimal Control Problems

A 2R robot moving along a given line

\[ |Q(t)| \leq Q_{Max} \]
Optimal Control Overview

**functional J**
\[ J(x) = \int_{t_0}^{t_f} f(x, \dot{x}, t) dt \]

**boundary conditions**
\[ x(t_0) = x_0, \quad x(t_f) = x_f \]

**minimizing path**
\[ x = x^*(t) \]

**for all continuous paths x(t)**
\[ J(x) \geq J(x^*) \]
\[ x(t) = x^* + \epsilon y(t) \quad \epsilon \ll 1 \]

\[ \Delta J = J(x^* + \epsilon y(t)) - J(x^*) = \epsilon V_1 + \epsilon^2 V_2 + O(\epsilon^3) \]

**first variation of J**
\[ V_1 = \int_{t_0}^{t_f} \left( y \frac{\partial f}{\partial x} + \dot{y} \frac{\partial f}{\partial \dot{x}} \right) dt \]

**second variation of J**
\[ V_2 = \int_{t_0}^{t_f} \left( y^2 \frac{\partial^2 f}{\partial x^2} + 2y\dot{y} \frac{\partial^2 f}{\partial x \partial \dot{x}} + \dot{y}^2 \frac{\partial^2 f}{\partial \dot{x}^2} \right) dt \]

**Euler-Lagrange equation.**
\[ \frac{\partial f}{\partial x} - \frac{d}{dt} \frac{\partial f}{\partial \dot{x}} = 0 \]
Example: Brachistochron

A curve joining points $P_1$ and $P_2$, and a frictionless sliding point.

discussed by Galilei in 1630
solved by Bernoulli in 1696

\[ J = \int_{t_0}^{t_f} dt \quad J = \int_1^2 \frac{ds}{v} \]

\[ v = \sqrt{2gy} \quad ds = \sqrt{1 + y'^2} \, dx \]

\[ J = \int_1^2 \sqrt{\frac{1 + y'^2}{2gy}} \, dx \]

Applying the Lagrange equations

\[ y \left(1 + y'^2\right) = 2r \]

The optimal curve is a **cycloid**

\[ x = r \left(\beta - \sin \beta\right) \]
\[ y = r \left(1 - \cos \beta\right) \]
Example: One DOF Dynamics System

\[ m\ddot{x} = g(x, \dot{x}) + f(t) \]

\[ f = m\ddot{x} - \mu mg \]

\[ |f(t)| \leq F \]
Example: Multi DOF Dynamics System

space station remote manipulator system
Example: Multi DOF Dynamics System
Example: Multi DOF Robotic Hand
Example: Multi DOF Stanford Arm
Pontryagin principle:

the optimal control input vector $Q(t)$ to minimize the time of motion of a multi DOF dynamic system between two given states on a prescribed path with bounded input

$$|Q(t)| \leq Q_{Max}$$

is the one that at every instant has at least one component saturated over the entire time interval.

$$Q_{Max} \text{ or } -Q_{Max}$$
Minimum Time and Bang-Bang Control

Consider a system with the following equation of motion:

\[ \dot{x} = f(x(t), Q(t)) \quad (14.1) \]

where \( Q \) is the control input, and \( x \) is the state vector of the system.

\[ x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix} \quad (14.2) \]

The minimum time problem is always subject to bounded input such as:

\[ |Q(t)| \leq Q_{Max} \quad (14.3) \]

The solution of the time-optimal control problem subject to bounded input is \textit{bang-bang control}. The control in which the input variable takes either the maximum or minimum values is called bang-bang control.
Floating Time Method

Time history of motion

Introducing two extra points, $x_{-1}$ and $x_{s+2}$, before the initial and after the final points.
Example: One DOF Dynamics System

\[ f_i = m \ddot{x}_i \]

\[ f = m \ddot{x} - \mu mg \]

\[ |f(t)| \leq F. \]

The rest conditions at the beginning

\[ \dot{x}_i = \frac{x_{i+1} - x_{i-1}}{\tau_i + \tau_{i-1}} \]

\[ x_{-1} = x_1 \]

\[ x_{s+2} = x_s \]

\[ \tau_0 = \tau_{-1} \]

\[ \tau_{s+1} = \tau_s. \]
Example: One DOF Dynamics System

at the initial point

\[ f_0 = m\ddot{x}_0 = \frac{4m}{2\tau_0^2} (x_1 - x_0) \]

The minimum value of the first floating time \( \tau_0 \) is found by setting \( f_0 = F \).

\[ \tau_0 = \sqrt{\frac{2m(x_1 - x_0)}{F}} \]

The minimum value of the final floating-time, \( \tau_s \), is achieved by \( f_{s+1} = -F \).

\[ \tau_s = \sqrt{\frac{2m(x_s - x_{s-1})}{-F}} \]

To find the minimum value of \( \tau_1 \)

\[ f_1 = \frac{4m}{\tau_1^2 + \tau_0^2} \left( \frac{\tau_0}{\tau_1 + \tau_0} x_2 + \frac{\tau_1}{\tau_1 + \tau_0} x_0 - x_1 \right) \]
Example: One DOF Dynamics System

\[ f_s \] can be found from the equation of motion at \( i = s \) and substituting \( \tau_s, \tau_{s-1}, x_{s-1}, x_s, \) and \( x_{s+1} \)

\[
f_s = \frac{4m}{\tau_s^2 + \tau_{s-1}^2} \left( \frac{\tau_{s-1}}{\tau_s + \tau_{s-1}} x_{s+1} + \frac{\tau_s}{\tau_s + \tau_{s-1}} x_{s-1} - x_s \right)
\]

If \( f_s \) does not break the constraint \( |f(t)| \leq F \), the problem is solved. However, it is expected that \( f_s \) breaks the constraint \( |f(t)| \leq F \). Now we reverse the procedure, and start a backward path.
Example: One DOF Dynamics System

\[ f = m\ddot{x} - \mu mg \]
\[ |f(t)| \leq F. \]

Rest-to-rest motion of a mass on a straight line time-optimally.

\[ f_i = m\ddot{x}_i \quad x(0) = 0 \quad v(0) = 0 \]
\[ x(t_f) = l \quad v(t_f) = 0. \]

\[ F = 10 \text{ N} \quad l = 1 \text{ m} \quad s + 1 = 200 \]

for \( \mu = 0 \) the switching point \( t = \tau = t_f/2 \) and

\[ f(t) = \begin{cases} 
  F & \text{if } t < \tau \\
  -F & \text{if } t > \tau.
\end{cases} \]
Example: One DOF Dynamics System

Graphical illustration of the floating time algorithm
Example: One DOF Dynamics System

Position history of the optimal force $f(t)$ for different friction coefficients $\mu$. 
Example: One DOF Dynamics System

Time history of the optimal input $f(t)$ for different friction coefficients $\mu$
Example: One DOF Dynamics System

Time history of the optimal motion $x(t)$ for different friction coefficients $\mu$. 

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>$t_f$</th>
</tr>
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<tr>
<td>0.6</td>
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<td>0</td>
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Algorithm

1. Divide the preplanned path of motion $x(t)$ into $s + 1$ intervals and specify all coordinate values $x_i$, ($i = 0, 1, 2, 3, ..., s + 1$)

2. Set $f_0 = +F$ and calculate $\tau_0 = \sqrt{\frac{2m(x_1 - x_0)}{F}}$

3. Set $f_{s+1} = -F$ and calculate $\tau_s = \sqrt{\frac{2m(x_s - x_{s-1})}{-F}}$

4. For $i$ from 1 to $s - 1$, calculate $\tau_i$ such that $f_i = +F$ and

   $$ f_i = m\ddot{x}_i - g(x_i, \dot{x}_i) $$

   $$ = \frac{4m}{\tau_i^2 + \tau_{i-1}^2} \left( \frac{\tau_{i-1}}{\tau_i + \tau_{i-1}} x_{i+1} + \frac{\tau_i}{\tau_i + \tau_{i-1}} x_{i-1} + x_i \right) - g(x_i, \dot{x}_i) $$

5. If $|f_i| \leq F$, then stop, otherwise set $j = s$.

6. Calculate $\tau_{j-1}$ such that $f_j = -F$

7. If $|f_{j-1}| \leq F$, then stop, otherwise set $j = j - 1$ and return to step 6
Convergence.

$\tau_i$ converges to the minimum possible value, as long as

$$\frac{\partial \dot{x}_i}{\partial \tau_i} < 0 \quad \text{and} \quad \frac{\partial \ddot{x}_i}{\partial \tau_{i-1}} > 0$$

$$Z_1 x_{s+1} + Z_2 x_s + Z_3 x_{s-1} < 0$$
$$Z_4 x_{s+1} + Z_5 x_s + Z_6 x_{s-1} > 0$$

for backward path

$$Z_1 x_{s+1} + Z_2 x_s + Z_3 x_{s-1} > 0$$
$$Z_4 x_{s+1} + Z_5 x_s + Z_6 x_{s-1} < 0$$

termination criterion

$$||f_i| - F| \leq \epsilon.$$
**Example: Beachistochrone**

\[ \begin{align*}
g & = 10 \text{ m/ s}^2 \\
s & = 100 \\
t_{f_1} & = 0.61084 \text{ s} \\
t_{f_2} & = 0.8057 \text{ s} \\
t_{f_1} & = 0.6176 \text{ s} \\
t_{f_2} & = 0.8077 \text{ s}
\end{align*} \]

An analytic solution shows

\[ \begin{align*}
x & = r (\beta - \sin \beta) \\
y & = r (1 - \cos \beta).
\end{align*} \]

Time optimal path for a falling unit mass from \( A(0, 1) \) to two different destinations.
Floating time technique for the $n$ DOF systems.

1. Divide the preplanned path of motion $x(t)$ into $s + 1$ intervals and specify all coordinate vectors $x_i$, $(i = 0, 1, 2, 3, \ldots, s + 1)$.

2. Develop the equations of motion at $x_0$ and calculate $\tau_0$ for which only one component of the force vector $f_0$ is saturated on its higher limit, while all the other components are within their limits.

\[
\begin{align*}
  f_{0_k} &= F_k, & k \in \{0, 1, 2, \cdots, n\} \\
  f_{0_r} &\leq F_r, & r = 0, 1, 2, \cdots, n, & r \neq k
\end{align*}
\]

3. Develop the equations of motion at $x_{s+1}$ and calculate $\tau_s$ for which only one component of the force vector $f_{s+1}$ is saturated on its higher limit, while all the other components are within their limits.

\[
\begin{align*}
  f_{s+1_k} &= -F_k, & k \in \{0, 1, 2, \cdots, n\} \\
  f_{s+1_r} &\leq F_r, & r = 0, 1, 2, \cdots, n, & r \neq k
\end{align*}
\]
Floating time technique for the $n$ DOF systems.

4. For $i$ from 1 to $s - 1$, calculate $\tau_i$ such that only one component of the force vector $\mathbf{f}_i$ is saturated on its higher limit, while all the other components are within their limits.

\[
\begin{align*}
    f_{ik} &= -F_k, & k &\in \{0, 1, 2, \ldots, n\} \\
    f_{ir} &\leq F_r, & r &\in \{0, 1, 2, \ldots, n\}, & r &\neq k
\end{align*}
\]

5. If $|\mathbf{f}_s| \leq F$, then stop, otherwise set $j = s$.

6. Calculate $\tau_{j-1}$ such that only one component of the force vector $\mathbf{f}_j$ is saturated on its lower limit, while all the other components are within their limits.

7. If $|\mathbf{f}_{j-1}| \leq F$, then stop, otherwise set $j = j - 1$ and return to step 6.
2R manipulator on a straight line.

A 2R planar manipulator with rigid arms

A 2R planar manipulator, moving from point (1, 1.5) to point (-1, 1.5) on a straight line $Y = 1.5$. 
equations of motion for 2R robotic manipulators

\[ P = A\ddot{\theta} + B\ddot{\phi} + C\dot{\theta}\dot{\phi} + D\dot{\phi}^2 + M \]
\[ Q = E\ddot{\theta} + F\ddot{\phi} + G\dot{\phi}^2 + N \]

where \( P \) and \( Q \) are the actuator torques and

\[ A = A(\varphi) = m_1 l_1^2 + m_2 \left( l_1^2 + l_2^2 + 2l_1 l_2 \cos \varphi \right) \]
\[ B = B(\varphi) = m_2 \left( l_2^2 + l_1 l_2 \cos \varphi \right) \]
\[ C = C(\varphi) = -2m_2 l_1 l_2 \sin \varphi \]
\[ D = D(\varphi) = m_2 l_1 l_2 \sin \varphi \]
\[ E = E(\varphi) = B \]
\[ F = m_2 l_2^2 \]
\[ G = G(\varphi) = -D \]
\[ M = M(\theta, \varphi) = (m_1 + m_2)gl_1 \cos \theta + m_2 gl_2 \cos (\theta + \varphi) \]
\[ N = N(\theta, \varphi) = m_2 gl_2 \cos (\theta + \varphi) \].
\begin{align*}
\nu(\dot{x}_i) &= \frac{x_{i+1} - x_{i-1}}{\tau_i + \tau_{i-1}} \\
\alpha(\ddot{x}_i) &= \frac{4}{\tau_i^2 + \tau_{i-1}^2} \left( \frac{\tau_{i-1}}{\tau_i + \tau_{i-1}} x_{i+1} + \frac{\tau_i}{\tau_i + \tau_{i-1}} x_{i-1} - x_i \right) \\
\dot{P}_i(t) &= A_i a(\dot{\theta}) + B_i a(\dot{\phi}) + C_i v(\dot{\theta}) v(\dot{\phi}) + D_i v^2(\dot{\phi}) + M_i \\
\dot{Q}_i(t) &= E_i a(\dot{\theta}) + F_i a(\dot{\phi}) + G_i v^2(\dot{\theta}) + N_i \\
|\dot{P}_i(t)| &\leq P_M, \quad |\dot{Q}_i(t)| \leq Q_M.
\end{align*}

\begin{align*}
\tau_0 &= \tau_1 \quad \theta_0 = \theta_2 \quad \phi_0 = \phi_2 \\
\tau_s &= \tau_{s+1} \quad \theta_s = \theta_{s+2} \quad \phi_s = \phi_{s+2}
\end{align*}

\begin{align*}
m_1 = m_2 &= 1 \text{ kg} \quad l_1 = l_2 = 1 \text{ m} \quad P_M = Q_M = 100 \text{ N m}
\end{align*}
Time optimal control inputs for a 2R manipulator moving on line $Y = 1.5$, $-1 < X < 1$
Illustration of motion of a 2R planar manipulator on line $y = 0$.

$m_1 = m_2 = 1\, \text{kg} \quad l_1 = l_2 = 1\, \text{m} \quad P_M = Q_M = 100\, \text{N}\,\text{m}$

$X(0) = 1.9\, \text{m} \quad X(t_f) = 0.5\, \text{m} \quad Y = 0$
Time optimal control inputs for a 2R manipulator moving on line $Y = 0$, $0.5 < X < 1.9$
Conclusion

Time optimal control of an n DOF dynamics system solution:

✓ At every instant of time, at least one actuator must be saturated while the others are within their limits.

✓ Floating-time method is to find the saturated actuator, the switching points, and the output of the non-saturated actuators.

✓ The floating-time method is based on discrete equations of motion, utilizing variable time increments.

✓ Smart equipment such as intelligent robotics in manufacturing processes, increase productivity, the robot should do its job in minimum time.
Biography of Presenter

Prof Reza Jazar (reza.Jazar@rmit.edu.au)
Professor of Mechanical Engineering (RMIT University)
Published more than 200 peer journal and conference paper and ten books.
Google scholar h-index 22 with 2700 citations.
Research interests: Nonlinear Dynamics and Vibrations, Robotics, MEMS
Thank You